SOLUTION TO PROBLEM #12128

Problem #12128. Proposed by O. Kouba, Syria. Let $F_n$ be the Fibonacci numbers, defined by $F_0 = 0$, $F_1 = 1$, and $F_{n+1} = F_n + F_{n-1}$ for $n \geq 1$. Find, in terms of $n$, the number of trailing zeros in the decimal representation of $F_n$.

Solution by Tewodros Amdeberhan and Victor H. Moll, Tulane University, New Orleans, LA, USA.

This is actually a consequence of the result in [1], i.e.

\[(1) \quad \nu_5(F_n) = \nu_5(n) \quad \text{and} \quad \nu_2(F_n) = \begin{cases} 1 & n \equiv 3 \mod 6 \\ \nu_2(4n) & n \equiv 0 \mod 6 \\ 0 & \text{otherwise} \end{cases} \]

where $\nu_p(\cdots)$ denotes the $p$-adic valuation function. The question at hand is simply the largest power of 10 that divides $F_n$, denote this by $c(n)$. Observe that 5 divides $F_n$ precisely when it divides $n$. Also 2 divides $F_n$ when 3 divides $n$. Therefore $c(n) = 0$ unless 15 divides $n$. Furthermore, $c(15x) = 1$ as soon as $x$ is odd due to (1). The remaining case is handled by

\[c(30y) = \min\{\nu_2(8y), \nu_5(5y)\}. \quad \square\]