Problem #12107. Proposed by C. I. Valean, Romania. Prove

\[ \int_0^1 \int_0^1 \frac{dx \, dy}{\sqrt{1 + x^2} \sqrt{1 + y^2} (1 - x^2 y^2)} = G \]

where \( G \) is the Catalan’s constant \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} \).

Solution by Tewodros Amdeberhan and Victor H. Moll, Tulane University, New Orleans, LA, USA. Start with the substitution \( x = \tan \theta \) and \( y = \tan \beta \) so that the given integral transforms into

\[ \int_0^{\pi/4} \int_0^{\pi/4} \frac{\cos \theta \cos \beta \, d\theta \, d\beta}{\cos^2 \theta \cos^2 \beta - \sin^2 \theta \sin^2 \beta} = \frac{1}{2} \int_0^{\pi/4} \log \left( \frac{\cos \theta + \sin \frac{\pi}{4}}{\cos \theta - \sin \frac{\pi}{4}} \right) d\theta \]

where the last equality is one of known integral representations of the Catalan’s constant. □